Théorie de la synchronisation - Equation de Duffing

Etude du régime transitoire - Graphes des courbes (X)

Caractéristiques du système

$T := 0.2 \cdot s$ $\qquad \omega_0 := \dfrac{2 \cdot \pi}{T}$ $\qquad J := 8 \cdot 10^{-7} \cdot kg \cdot m^2$ $\qquad q_0 := 270 \cdot deg$

Frottement visqueux

$\eta := 0.002$ $\qquad C := 2 \cdot J \cdot \eta \cdot \omega_0$ $\qquad F_{v\_max} := C \cdot \omega_0 \cdot q_0$ $\qquad \lambda := \dfrac{F_{v\_max}}{J \cdot \omega_0^2}$ $\qquad h := 2 \cdot \dfrac{\eta}{\lambda}$

$F_{v\_max} = 0.015\, N \cdot mm$ $\qquad \lambda = 0.019$ $\qquad h = 0.212$

Frottement quadratique

$B := 0.05 \cdot F_{v\_max}$ $\qquad \beta_1 := \dfrac{B}{\lambda \cdot J \cdot \omega_0^2}$ $\qquad \beta_1 = 0.05$ $\qquad F_{q\_max} := B \cdot q_0^3$ $\qquad F_{q\_max} = 0.078\, N \cdot mm$

Excitation harmonique

$A_c := \dfrac{8 \cdot h^3}{3\sqrt{3}}$ $\qquad A_c = 0.015$ $\qquad a_c := \sqrt{\dfrac{4 \cdot A_c}{3 \cdot \beta_1}}$ $\qquad a_c = 0.626$

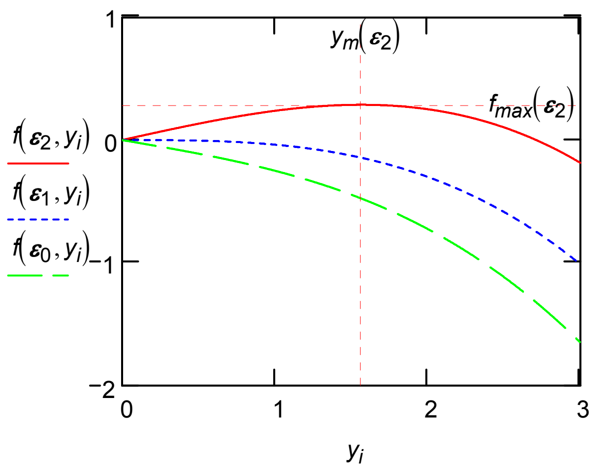
$F_{harm} := a_c \cdot \left(\lambda \cdot J \cdot \omega_0^2\right)$ $\qquad F_{harm} = 9.322 \times 10^{-3}\, N \cdot mm$

Etude des fonctions f(y)

$n := 1000$ $\qquad i := 0 .. n$ $\qquad \Delta y := \dfrac{3}{n}$ $\qquad y_i := i \cdot \Delta y$

$f(\varepsilon, y) := \varepsilon \cdot y - \dfrac{3}{4} \cdot \beta_1 \cdot y^3$ $\qquad \boldsymbol{\varepsilon} := (-h \quad 0 \quad 1.3 \cdot h)^T$ $\qquad \boldsymbol{\varepsilon}^T = (-0.212 \quad 0 \quad 0.276)$

$y_m(\varepsilon) := \dfrac{2}{3} \cdot \sqrt{\dfrac{\varepsilon}{\beta_1}}$ $\qquad y_m(\boldsymbol{\varepsilon}_2) = 1.566$ $\qquad f_{max}(\varepsilon) := \dfrac{4}{9} \cdot \varepsilon \cdot \sqrt{\dfrac{\varepsilon}{\beta_1}}$ $\qquad f_{max}(\boldsymbol{\varepsilon}_2) = 0.288$



Graphes des courbes (X)

Courbes en arche simple

$x_0 := 0 \qquad x_1 := 2\cdot\pi \qquad \Delta x := \dfrac{x_1 - x_0}{n} \qquad x_i := x_0 + i\cdot\Delta x$

$$X1_a := \left| \begin{array}{l} \text{for } i \in 0..n \\ \quad \left| \begin{array}{l} Z \leftarrow polyracines\left(\left(-a_c\cdot\cos(x_i) \quad \varepsilon_0 \quad 0 \quad \dfrac{-3}{4}\cdot\beta_1\right)^T\right) \\ \text{for } j \in 0..2 \\ \quad X_{i,j} \leftarrow Z_j\cdot(Im(Z_j) = 0)\cdot(Re(Z_j) > 0) \end{array} \right. \\ X \end{array} \right.$$

$$X2_a := \left| \begin{array}{l} \text{for } i \in 0..n \\ \quad \left| \begin{array}{l} Z \leftarrow polyracines\left(\left(-a_c\cdot\cos(x_i) \quad 0 \quad 0 \quad \dfrac{-3}{4}\cdot\beta_1\right)^T\right) \\ \text{for } j \in 0..2 \\ \quad X_{i,j} \leftarrow Z_j\cdot(Im(Z_j) = 0)\cdot(Re(Z_j) > 0) \end{array} \right. \\ X \end{array} \right.$$

Courbe en arche cintrée

$a_1 := 2\cdot f_{max}(\varepsilon_2)$

$x_{min} := \arccos\left(\dfrac{f_{max}(\varepsilon_2)}{a_1}\right)$

$x_{max} := 2\cdot\pi - x_{min}$

$x_{min} = 1.047 \qquad x_{max} = 5.236$

$$X_{ac} := \left| \begin{array}{l} \text{for } i \in 0..n \\ \quad \left| \begin{array}{l} Z \leftarrow polyracines\left(\left(-a_1\cdot\cos(x_i) \quad \varepsilon_2 \quad 0 \quad \dfrac{-3}{4}\cdot\beta_1\right)^T\right) \\ \text{for } j \in 0..2 \\ \quad X_{i,j} \leftarrow Z_j\cdot(Im(Z_j) = 0)\cdot(Re(Z_j) > 0) \end{array} \right. \\ X \end{array} \right.$$

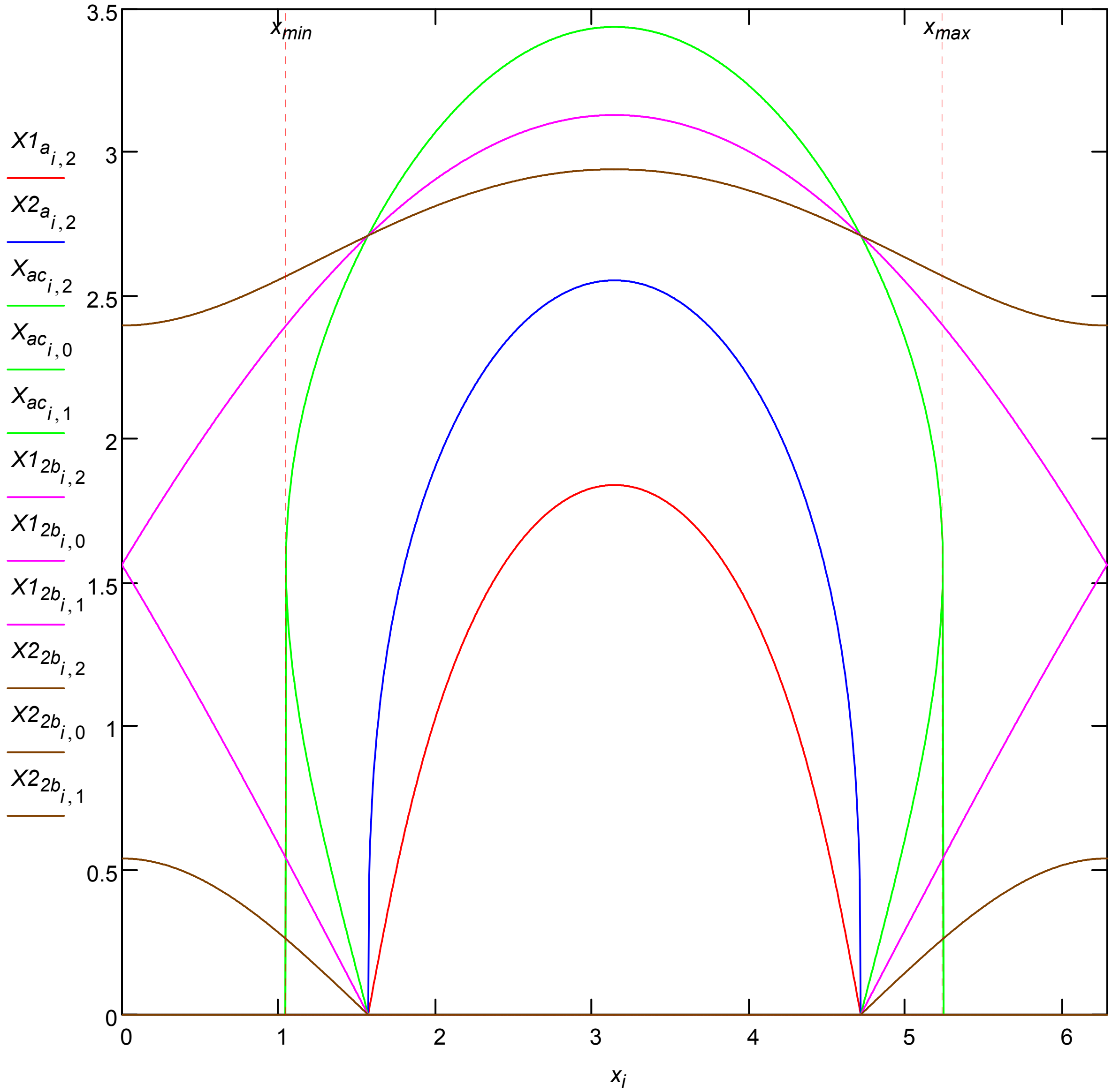
Courbes à deux branches

$a_1 := f_{max}(\varepsilon_2)$

$$X1_{2b} := \left| \begin{array}{l} \text{for } i \in 0..n \\ \quad \left| \begin{array}{l} Z \leftarrow polyracines\left(\left(-a_1\cdot\cos(x_i) \quad \varepsilon_2 \quad 0 \quad \dfrac{-3}{4}\cdot\beta_1\right)^T\right) \\ \text{for } j \in 0..2 \\ \quad X_{i,j} \leftarrow Z_j\cdot(Im(Z_j) = 0)\cdot(Re(Z_j) > 0) \end{array} \right. \\ X \end{array} \right.$$

$a_1 := 0.5\cdot f_{max}(\varepsilon_2)$

$$X2_{2b} := \left| \begin{array}{l} \text{for } i \in 0..n \\ \quad \left| \begin{array}{l} Z \leftarrow polyracines\left(\left(-a_1\cdot\cos(x_i) \quad \varepsilon_2 \quad 0 \quad \dfrac{-3}{4}\cdot\beta_1\right)^T\right) \\ \text{for } j \in 0..2 \\ \quad X_{i,j} \leftarrow Z_j\cdot(Im(Z_j) = 0)\cdot(Re(Z_j) > 0) \end{array} \right. \\ X \end{array} \right.$$


$X1_{a_{i,2}}$
$X2_{a_{i,2}}$
$X_{ac_{i,2}}$
$X_{ac_{i,0}}$
$X_{ac_{i,1}}$
$X1_{2b_{i,2}}$
$X1_{2b_{i,0}}$
$X1_{2b_{i,1}}$
$X2_{2b_{i,2}}$
$X2_{2b_{i,0}}$
$X2_{2b_{i,1}}$
x_{min}
x_{max}
3.5
3
2.5
2
1.5
1
0.5
0
0
1
2
3
4
5
6
x_i